UNIFORM BOUNDARY LAYER SUCTION IN THE SLIP REGIME

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It is shown that in the presence of uniform suction in the boundary layer the slip velocity and temperature jump should be taken into account irrespective of the degree of rarefaction.

The problem of uniform boundary-layer suction on a flat plate was examined in [1–3]. The existence of a constant negative velocity at the plate surface makes it possible to obtain a solution that does not depend on the longitudinal x-coordinate. This solution is applicable only at a certain distance from the leading edge. This asymptotic state is reached when $x_{ac} = 4\mu u_\infty/\rho v_0^2$ [1]. A similar problem for a nonisothermal rarefied gas flow was solved in [4]; here, the usual slip and temperature-jump equations were used as the boundary conditions

$$u = \frac{2-f}{f}\, l\, \frac{du}{dy}, \tag{1}$$

$$T - T_0 = \frac{15(2-f)}{8f}\, l\, \frac{dT}{dy}. \tag{2}$$

However, in the case of a permeable surface [5] condition (2) changes (it includes an additional term containing the suction velocity v_0).

We found an asymptotic solution for the problem of uniform suction in a rarefied gas boundary layer with allowance for these modifications.

Assume that the gas is incompressible and that the specific heat, coefficient of viscosity, and Prandtl number are constant (valid for small temperature drops and small Mach numbers M_∞).

From the continuity equation

$$v = v_0 = \text{const} < 0, \tag{3}$$

while the equations of motion and energy take the form

$$\rho v_0 \frac{du}{dy} = \mu \frac{d^2u}{dy^2},$$

$$\rho v_0 \frac{dT}{dy} = \frac{\mu}{\sigma}\frac{d^2T}{dy^2} + \frac{\mu}{c_p}\left(\frac{du}{dy}\right)^2. \tag{4}$$

For the boundary conditions at the surface of the plate (y = 0) we use condition (1) and the temperature jump equation from [5]

$$T - T_0 = \frac{2-f}{f}\left[\frac{15}{8}\, l\, \frac{dT}{dy} - \left(\frac{2\pi T_0}{R}\right)^{1/2}\frac{v_0}{8}\right]. \tag{5}$$

In the flow outside the boundary layer

$$u = u_\infty, \quad T = T_\infty. \tag{6}$$

We introduce the new dimensionless variable

$$z = \frac{\rho |v_0|}{\mu}\, y.$$

Now Eqs. (3) and (4) may be written as

$$\frac{d^2u}{dz^2} + \frac{du}{dz} = 0, \tag{7}$$

$$\frac{d^2T}{dz^2} + \sigma\frac{dT}{dz} = -\frac{\sigma}{c_p}\left(\frac{du}{dz}\right)^2, \tag{8}$$

and boundary conditions (1) and (5) at z = 0 take the following form:

$$u = h\frac{du}{dz}, \tag{9}$$

$$T - T_0 = \frac{15}{8}\, h\, \frac{dT}{dz} + \frac{2-f}{f}\left(\frac{2\pi T_0}{R}\right)^{1/2}\frac{|v_0|}{8}, \tag{10}$$

where

$$h = \frac{2-f}{f}\,\frac{\rho l |v_0|}{\mu}.$$

Since the dimensionless parameter h does not depend on temperature, Eq. (7), the first of conditions (6), and condition (9) completely determine the velocity profile

$$u = u_\infty\left(1 - \frac{\exp\{-z\}}{1+h}\right). \tag{11}$$

The solution of energy equation (8) with the second of conditions (6) is

$$T = T_\infty - \frac{\sigma u_\infty^2}{2c_p(2-\sigma)(1+h)^2}\times$$
$$\times\exp\{-2z\} + A\exp\{-\sigma z\}, \tag{12}$$

Slip Velocity and Temperature Jump as Functions of the Parameter h

h	$\lvert v_0\rvert$, m/sec	u(0), m/sec	T(0), °K	ΔT, °K from (5)	Δ'T, °K from (2)	(ΔT - Δ'T), °K
0.01	2.6	2.0	352.2	2.2	1.3	0.9
0.05	13.1	9.5	360.0	10.0	5.9	4.1

where the coefficient A is found from (10)

$$A = \frac{T_0 - T_\infty}{1 + \frac{15}{8} h\sigma} + \frac{\sigma u_\infty^2 \left(1 + \frac{15}{4} h\right)}{2(1+h)^2 c_p (2-\sigma)\left(1 + \frac{15}{8} h\sigma\right)} + \frac{2-f}{8f}\left(\frac{2\pi T_0}{R}\right)^{1/2} \frac{|v_0|}{1 + \frac{15}{8} h\sigma}. \quad (13)$$

Setting z = 0 in (11) and (12), we find the slip velocity and the gas temperature at the surface

$$u(0) = u_\infty \frac{h}{1+h}, \quad (14)$$

$$T(0) = T_\infty + \frac{T_0 - T_\infty}{1 + \frac{15}{8} h\sigma} + \frac{15 h \sigma u_\infty^2}{16 c_p (1+h)^2 \left(1 + \frac{15}{8} h\sigma\right)} + \frac{2-f}{8f}\left(\frac{2\pi T_0}{R}\right)^{1/2} \frac{|v_0|}{1 + \frac{15}{8} h\sigma}. \quad (15)$$

From (14) and (15) we conclude that for given σ, f, u_∞, T_0, and T_∞ the slip velocity and temperature jump are determined by the parameter h. It is known from elementary kinetic theory that $\mu = \rho l a/2$. Consequently, $h = (2(2-f)/f)(|v_0|/a)$, i.e., for given f, the parameter h is proportional to the relative suction velocity and does not depend on the degree of rarefaction. Hence, other things being equal, the slip velocity and temperature jump depend only on the suction velocity. In this case the boundary conditions must be written in the form of (1) and (5) for any degree of rarefaction. In other words, the usual boundary conditions of no slip and equality of the gas and wall temperatures do not hold.

The last term in expression (15) does not depend on the free-stream parameters. However, as M_∞ increases, the relative contribution of this term to the temperature jump decreases.

The table gives the results of numerical calculations for air at h = 0.01, and 0.05 for the following values of the parameters: $\sigma = 1$, $f = 1$, $u_\infty = 200$ m/sec, $T_0 = 350°$, $T_\infty = 400°$.

As seen from the table (last column), the second term in condition (5) makes an additional contribution to the temperature jump at the wall which coincides in order of magnitude with the temperature jump calculated using condition (2).

Using (11), we obtain an expression for the shear stress on the plate and the displacement thickness of the boundary layer

$$\tau = \mu \left.\frac{du}{dy}\right|_{y=0} = \frac{\rho |v_0| u_\infty}{1+h},$$

$$\delta^* = \int_0^\infty \left(1 - \frac{u}{u_\infty}\right) dy = \frac{\mu}{\rho |v_0|} \frac{1}{1+h}.$$

When slip is not accounted for in the boundary conditions, these quantities are, respectively, given by

$$\tau_0 = \rho |v_0| u_\infty, \quad \delta_0^* = \mu/\rho |v_0|.$$

Hence we conclude that taking slip into account in the boundary conditions leads to a reduction in skin friction and displacement thickness of the boundary layer.

In determining the specific heat flux at the wall in a flow with slip it is necessary to consider the heat supplied to the surface as a result of the work done by the friction forces

$$q = \left(\lambda \frac{dT}{dy} + \mu u \frac{du}{dy}\right)_{y=0}.$$

Using (11) and (12), we obtain

$$q = -A c_p \rho |v_0| + \frac{\rho |v_0| u_\infty^2}{(2-\sigma)(1+h)^2}[1 + (2-\sigma)h]. \quad (16)$$

If the slip and temperature jump are not taken into account, the specific heat flux is

$$q_0 = c_p \rho |v_0| \left(T_\infty - T_0 + \frac{u_\infty^2}{2 c_p}\right). \quad (17)$$

It should be noted that τ, δ^*, and q depend not only on h (or $|v_0|$), but also on ρ, i.e., on the degree of rarefaction. However, the relative changes in these quantities do not depend on ρ. In particular, as calculations show, taking the slip and temperature-jump conditions into account leads to a reduction in the specific heat flux q as compared with q_0 by about 14% at h = 0.05 and 3% at h = 0.01.

The parameter h characterizes the degree of rarefaction of the gas only if $\rho |v_0|$ = const. Then $h \sim l$ and decreases with increase in gas density. On the other hand, at constant $\rho |v_0|$ an increase in gas density causes a decrease in suction velocity. Therefore in the limit as $l \to 0$ we obtain the solution of the familiar problem of uniform suction without allowance for slip and temperature jump [3].

NOTATION

ρ is the gas density; v_0 is the suction velocity; u is the tangential component of gas flow velocity; T is the gas temperature; T_0 is the plate temperature; $\Delta T = T(0) - T_0$; f is the accommodation factor; l is the mean free path; μ and λ are the coefficients of viscosity and thermal conductivity of the gas, respectively; c_p is the specific heat at constant pressure; σ is the Prandtl number; a is the mean thermal velocity; R = k/m; k is the Boltzmann constant; m is the molecular mass.

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